Adaptive Learning on Time Series: Method and Financial Applications

Parley Ruogu Yang[#†*], Ryan Lucas[‡*], Camilla Schelpe[*]

[#] Correspondent parleyyang.github.io [†] CCIMI, Faculty of Mathematics, University of Cambridge

[‡] University College Dublin [*] Optimal Portfolio Research Group optimalportfolio.github.io

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Background

- Time-varying distribution: $y_t \sim P^t_{Y|X}(\cdot|x_t;\theta)$ for all $t \in [T]$. We have access to data $\{(x_t, y_t) : t \in [T]\}$, but notice there can be many cases where $P^t_{Y|X} \neq P^{t+1}_{Y|X}$. This induces many estimation methods attempting to pick out change-points (engineering), time-varying parameters and moment estimators (time series and financial econometrics), and attempts in robust statistics. Modern ML architectures like RNN also aim to capture this.
- Associated problems:
  - Modelling & Forecast: which model and estimation method shall be used?
  - Evaluation: how to evaluate the performance of models under time-varying distribution?

Financial example: VIX, Yield Curve, and and S&P 500 during 2020

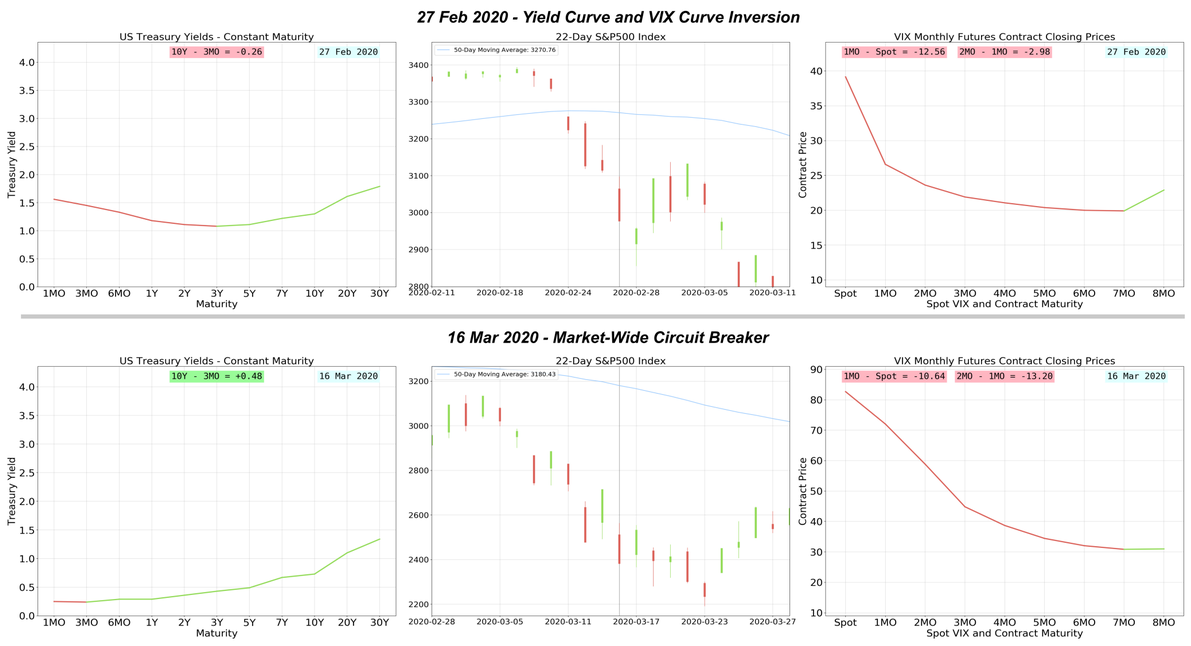
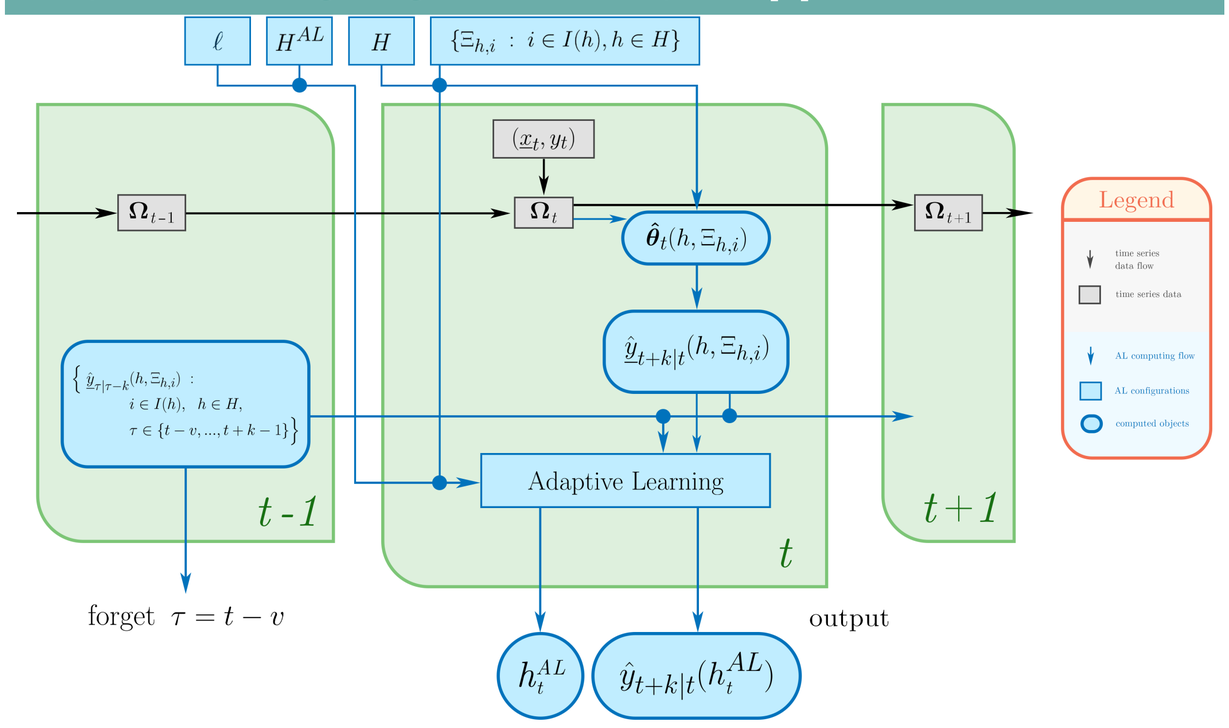


Figure: The February 2020 US Treasury yield curve inversion, coupled with a negatively sloping VIX futures term structure, preceded the pandemic-induced market crash of March 2020 (above). During the crisis, the VIX term structure exhibited extreme inversion, then accompanied by a drop in the overall level of Treasury yields (below).

Adaptive Learning: Algorithmic viewpoint [4]



Examples of loss functions

- Basic forms used in [2, 3, 4]:

$$\ell^{\texttt{Norm, single-valued}}(h, \Xi_{h,i}; \lambda, p) := \sum_{\tau=t-v+1}^{t} \lambda^{t-\tau} |\hat{y}_{\tau|\tau-k} - y_\tau|^p$$

  where v is a hyperparameter for window size
- Basic forms with tricks used in [4]:

$$\ell^{\texttt{Norm, multi-valued}}(h, \Xi_{h,i}; \lambda, p) := \sum_{\tau=t-v+1}^{t} \lambda^{t-\tau} ||\hat{\boldsymbol{y}}_{\tau|\tau-k} - y_\tau \mathbf{1}_k||_p^p$$

  where $\hat{\boldsymbol{y}}_{\tau|\tau-k} = (\hat{y}_{\tau|\tau-1}, \hat{y}_{\tau|\tau-2}, ..., \hat{y}_{\tau|\tau-k})$
- Interesting Penalisation used in [3]:

$$\ell^{\texttt{Penalised}}(h, \Xi_{h,i}; \lambda, p) := \ell^{\texttt{Global}}(h, \Xi_{h,i}; \lambda, p) + D(h, h^*_{t-1})$$

  where D measures distance, e.g. in terms of complexity and parametric dimensions

Empirical results: improved forecasts and financial portfolio with interpretation

k	Type	Configuration	1000 × MSE	CS	SR	100 × ANR	MDD
3	AL	Ensemble-MC	**2.984**	**0.500**	**-0.958**	**-51.591**	**-0.220**
	Fixed	MG3T, AR(4), $w = 252$, VIX 3-8m	4.102	0.409	-2.081	-81.887	-0.224
5	AL	Ensemble-MC	**7.067**	0.515	**-1.415**	**-70.035**	**-0.214**
	Fixed	MG3N, $w = 252$, VIX 3-6m	11.526	**0.529**	-2.264	-100.219	-0.288

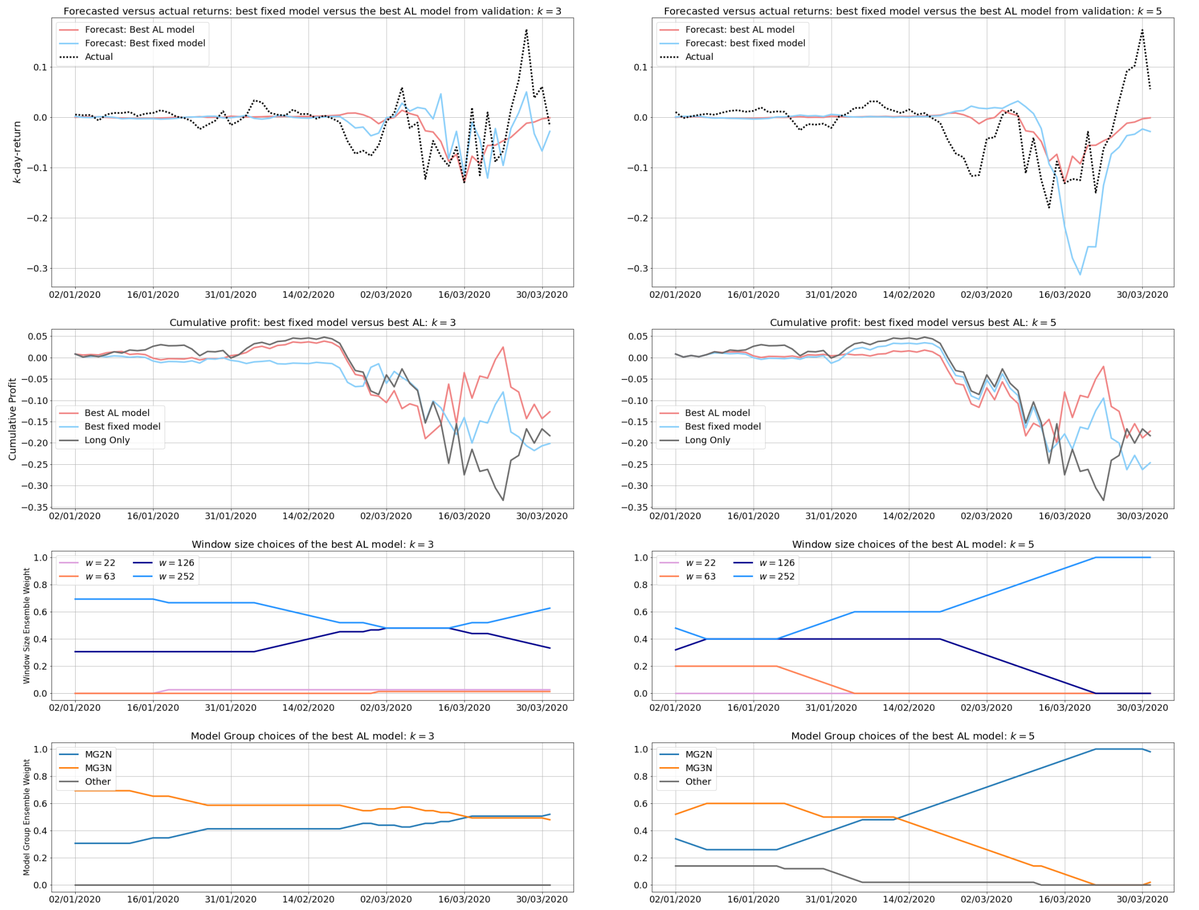
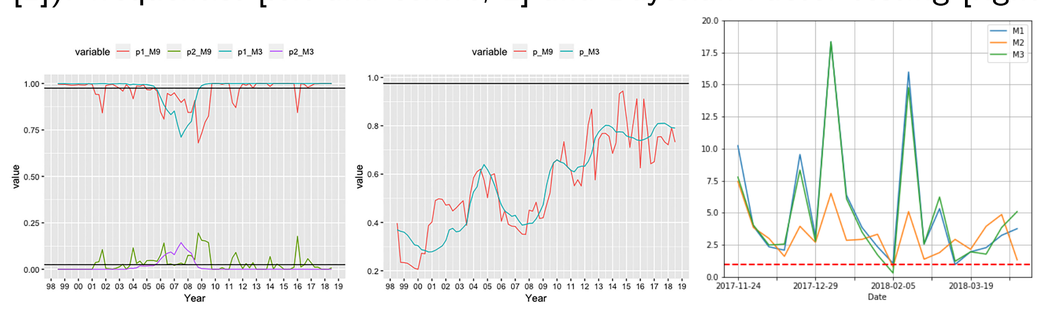


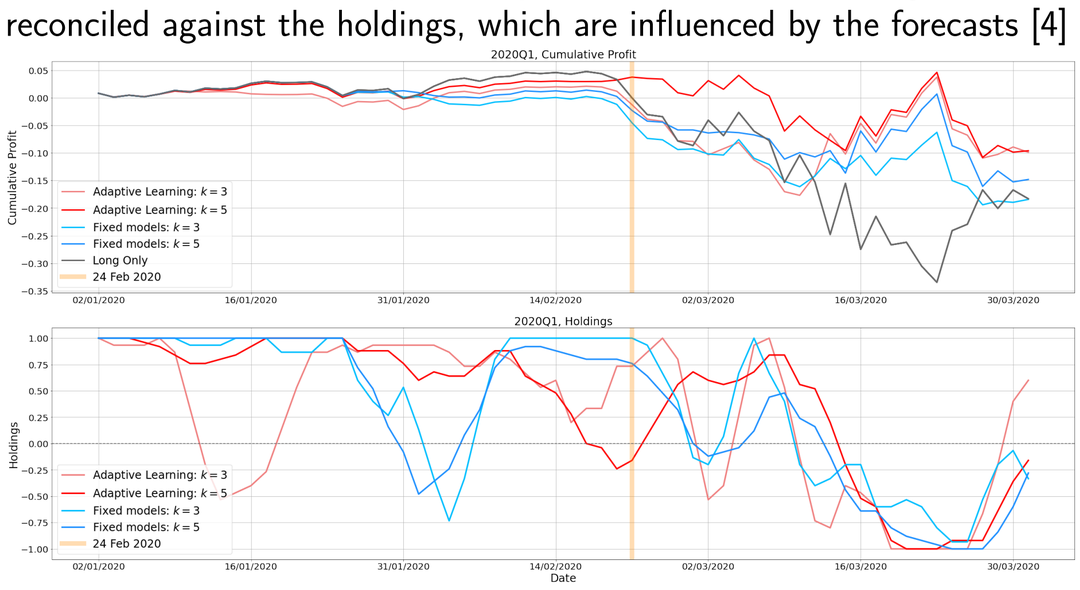
Figure: 2020 Q1 testing performance with models selected based on data in 2019 [4].

Extensions: interpretability, statistical testing, and financial portfolio management

- Interpretability due to direct access to model formations (e.g. panels 4 and 5 in figure above)
- Associated testing for parameters and models (extension to policy formations [1]): frequentist [left and centre, 2] and Bayesian Factor testing [right, 3]



- Financial decision based upon AL: profit and loss from trading can be reconciled against the holdings, which are influenced by the forecasts [4]



References

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